Supersymmetric Phase Transitions and Gravitational Waves at LISA

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Gravitational waves generated during a first-order electroweak phase transition have a typical frequency which today falls just within the band of the planned space interferometer LISA. Contrary to what happens in the Standard Model, in its supersymmetric extensions the electroweak phase transition may be strongly first order, providing a mechanism for generating the observed baryon asymmetry in the Universe. We show that during the same transition the production of gravitational waves can be rather sizable. While the energy density in gravitational waves can reach at most $h_0^2 \Omega_{\rm gw} \simeq 10^{-16}$ in the Minimal Supersymmetric Standard Model, in the Next-to-Minimal Supersymmetric Model, in some parameter range, $h_0^2 \Omega_{\rm gw}$ can be as high as 4×10^{-11}. A stochastic background of gravitational waves of this intensity is within the reach of the planned sensitivity of LISA. Since in the Standard Model the background of gravitational waves is totally negligible, its detection would also provide a rather unexpected experimental signal of supersymmetry and a tool to descriminate among supersymmetric models with different Higgs content.

During its evolution the Universe has probably undergone a series of phase transitions and some important remnants of these events may exist today, including the observed baryon asymmetry of the Universe and a stochastic background of gravitational waves (GWs). During a first-order phase transition the Universe is "trapped" in a metastable state – the false vacuum – which is separated from the true vacuum by a barrier in the potential of the order parameter, usually a scalar field ϕ. The transition takes place through the nucleation of bubbles of the new phase and most of the latent heat released in the transition is converted into kinetic energy that makes the bubbles expand. When the bubble walls collide particle production takes place [1] and part of the energy is radiated into gravitational waves [2–9]. The properties of this gravitational radiation – such as the characteristic frequency and the intensity – depend on the typical energy scales involved in the transition. In particular, the electroweak phase transition is expected to produce a background of GWs with a peak frequency around the milliHertz. This frequency happens to be the range most relevant for the space interferometer LISA [10], which is planned to fly by 2010.

A strongly first order phase transition could also provide a mechanism for generating the observed baryon asymmetry in the Universe [11]. For such a reason, the strength of the phase transition has been investigated in details in the Standard Model (SM) and in extensions of it. Unfortunately, non-perturbative results have revealed that there is no hot electroweak phase transition in the Standard Model for Higgs masses larger than m_W [12]. Therefore, GWs are not produced at the SM electroweak transition.

Among the possible extensions of the SM at the weak scale, its supersymmetric extensions are the best moti-
vated ones. In the Minimal Supersymmetric Standard Model (MSSM), a strong enough phase transition requires light Higgs and stop eigenstates. For a Higgs mass in the range $(110 - 115)$ GeV, there is a window in the right-handed stop mass $m_{\rm stop}$ in the range $(105 - 165)$ GeV [13]. If the Higgs is heavier than about 115 GeV, stronger constraints are imposed on the space of supersymmetric parameters. However, the strength of the transition can be further enhanced in extensions of the MSSM, for instance with the addition of a gauge singlet in the Higgs sector [14].

The goal of this paper is to compute the amount of gravitational waves generated during the electroweak phase transition in supersymmetric extensions of the SM. We will find that, depending on the model and on the region of parameter space, the stochastic background of GWs generated in the collision of bubbles nucleated during the supersymmetric electroweak phase transition can be within the sensitivity of the planned space-interferometer LISA. This opens the exciting possibility that the very same supersymmetric physics responsible for the generation of the primordial baryon asymmetry is also able to provide us with a detectable background of gravitational waves.

A stochastic background of GWs [4,15,16] can be characterized by the dimensionless quantity

$$\Omega_{\rm gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{\rm gw}}{d\log f}, \tag{1}$$

where $\rho_{\rm gw}$ is the energy density associated to GWs, f is their frequency and ρ_c is the present value of the critical energy density, $\rho_c = 3H_0^2/(8\pi G_N)$, with $H_0 = 100 h_0$ Km/sec Mpc; h_0 parametrizes the uncertainty in H_0. In fact, it is more convenient to characterize the stochastic background of GWs with the quantity $h_0^2 \Omega_{\rm gw}(f)$, which

is independent of h_0. In Fig. 1 we show the most relevant bounds on cosmological stochastic GW backgrounds, together with the experimental sensitivities of the various detectors under construction, as discussed in Refs. [16,17]. In particular, LISA is expected to reach a sensitivity of order $h_0^2 \Omega_{\rm gw} \simeq 10^{-12}$ at $f = 1$ mHz. At this frequency – however – a cosmological signal could be masked by an astrophysical background due to unresolved compact white dwarf binaries. Its strength is uncertain, since it depends on the rate of white dwarf mergers and it is estimated to be $h_0^2 \Omega_{\rm gw} \simeq 10^{-11}$ [10]. At a frequency $f \simeq 10$ mHz the LISA sensitivity is expected to be of order $h_0^2 \Omega_{\rm gw} \simeq 10^{-11}$, and the astrophysical background is below this value. (Correlating two detectors one usually gains many orders of magnitudes in the sensitivity, but because of this extragalactic background, even if one would fly two LISA detectors, the sensitivity of the correlation would be limited to $h_0^2 \Omega_{\rm gw} \simeq 5 \times 10^{-13}$ [18]).
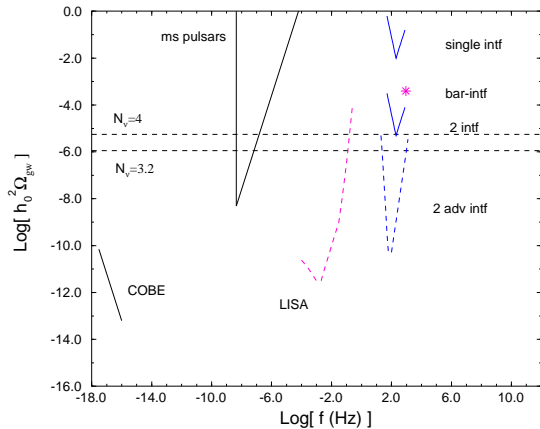

FIG. 1. The bounds from nucleosynthesis (horizontal dashed lines, for $N_\nu = 4$ and for $N_\nu = 3.2$), from COBE and from ms pulsars, together with the sensitivity of ground based detectors and of LISA. See ref. [17] for details.

Two are the basic quantities which play a role in the determination of the GW background generated during a first-order phase transition. The parameter α gives a measure of the jump in the energy density experienced by the order parameter ϕ during the transition from the false to the true vacuum and it is the ratio between the false vacuum energy density and the energy density of the radiation at the transition temperature T_*. The parameter β characterizes the bubble nucleation rate per unit volume, which can be expressed as $\Gamma = \Gamma_0 \exp(\beta t)$ [7]. Thus $\beta^{-1} = \Gamma/\dot{\Gamma}$ represents roughly the duration of the phase transition and the characteristic frequency of the gravitational radiation is expected to be $2\pi f \simeq \beta$. The stronger is the transition, the larger is α and the smaller is β because of the larger amount of supercooling experienced by the system before the transition. The present (*i.e.* properly red-shifted) peak frequency of the GW background is determined by β and by H_*, T_*, g_*, which are respectively the Hubble parameter, the temperature and the number of relativistic degrees of freedom at the

moment of the transition [9],

$$f_{\rm peak} \simeq 5.2 \times 10^{-8} \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{1 \text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} \text{ Hz .} \quad (2)$$

Typical values for β/H_* for the electroweak phase transition are between 10^2 and 10^3, with $T_* = \mathcal{O}(100)$ GeV. This gives a frequency $f_{\rm peak}$ in the range $(10^{-4} - 5 \times 10^{-3})$ Hz. From Fig. 1 we see that this is precisely the range in which LISA achieves its maximum sensitivity.

The intensity of the radiation produced is given by [9]

$$h_0^2 \Omega_{\rm gw} \simeq 10^{-6} \left(\frac{0.7\,\alpha + 0.2\sqrt{\alpha}}{1 + 0.7\,\alpha} \times \frac{\alpha}{1 + \alpha}\right)^2 \times$$
$$\times \left(\frac{H_*}{\beta}\right)^2 \left(\frac{v^3}{0.25 + v^3}\right) \left(\frac{100}{g_*}\right)^{1/3}, \quad (3)$$

where v is the velocity at which the bubble expands. In the following we will use a value of the velocity $v = v(\alpha)$ as given in Ref. [9] for bubble detonation. Eq. (3) makes it clear that – in order to produce a relevant signal – one needs large α and small β, *i.e.* a strongly first-order transition.

Before launching ourselves into details, let us briefly describe how we have computed the relevant quantities appearing in Eqs. (2) and (3), namely T_*, α and β/H_*, once the thermal effective potential $V(\phi, T)$ for the Higgs scalar field(s) is given. The rate of tunneling per unit volume from the metastable minimum to the stable one is suppressed by the exponential of an effective action $\Gamma = \Gamma_0 \, e^{-\frac{S_3(T)}{T}}$, where Γ_0 is of the order of T^4, and S_3 is the extremum of the spatial Euclidean action $S_3(T) = \int d^3 x \left[\frac{1}{2} (\vec{\nabla} \phi_b)^2 + V(\phi_b, T)\right]$ computed for the configuration of the scalar field(s) describing the bubble wall which interpolates between the false and the true vacuum. The transition takes place when the probability of nucleating a single bubble within one horizon volume becomes $\mathcal{O}(1)$. This condition fixes T_* to satisfy $S_3(T_*)/T_* \simeq \ln (M_{\rm Pl}/100 \text{ GeV})^4 \simeq 140$ for the electroweak transition. The parameter α is readily computed from the definition given above, while β/H_* is given by

$$\frac{\beta}{H_*} = T_* \left. \frac{d\,(S_3/T)}{dT}\right|_{T_*}. \quad (4)$$

As we have already noticed, extensions of the SM are required to obtain a first-order phase transition at the electroweak scale.

In the MSSM two complex Higgs doublets H_1 and H_2 are present in the Higgs sector and new CP-violating phases appear which can drive enough amount of baryon asymmetry. If the mass m_A of the CP-odd field in the Higgs sector is much larger than m_W, only one light Higgs scalar ϕ is left and its potential is similar to the one in the SM. When $m_A \sim m_W$ the full two-Higgs potential should be considered, but the strength of the phase transition is weakened [13]. The one-loop thermal corrections

to the effective potential make the quadratic term positive at high temperature and create a negative cubic term due to loops of the massive bosons in the theory. Due to the presence of this cubic term and the positivity of the quadratic one, there exists a range of temperature in which the point $\phi = 0$ is a local minimum separated from the true simmetry-breaking one by a small potential barrier, that is precisely the set-up for a first-order phase transition. The strength of the phase transition is enhanced by the presence of new bosons coupled to the Higgs, a significant role being played by the right-handed stop, which is – apart from the Higgs itself – the lightest scalar in the theory and has the largest Yukawa coupling to the Higgs ϕ.

To study the amount of gravitational waves generated during the electroweak phase transition within the MSSM we have made use of the thermal potential corrected up to two-loop level. Indeed, two-loop corrections have been shown to render the phase transition significantly stronger in the MSSM [13]. The most relevant parameters in the game are the Higgs mass m_{higgs}, the right-handed stop mass m_{stop} and the zero temperature ratio between the vacuum expectation values of the two neutral Higgses $\tan\beta_{\text{MSSM}} = \langle H_2 \rangle / \langle H_1 \rangle$.

Our strategy has been the following. For any given choice of the parameters of the model, we have first numerically computed the nucleation temperature T_* by imposing that, for the Higgs field configuration describing the nucleated bubble, the condition $S_3(T_*)/T_* \simeq 140$ is satisfied. Then we have computed the parameters α and β through Eq. (4). Our results are summarized in Figs. 2 and 3.
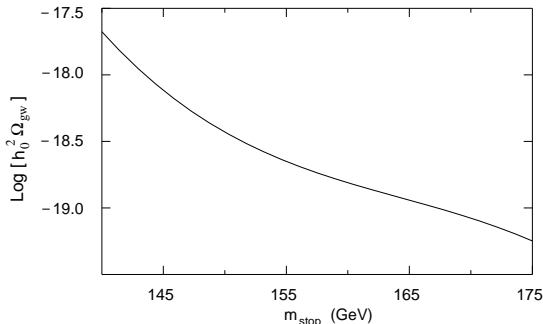
FIG. 2. $h_0^2 \Omega_{\text{gw}}$ as a function of the stop mass for a 110 GeV Higgs mass.

The general prediction is that the intensity of the gravitational waves produced during the MSSM phase transition is too small for LISA. For instance, taking a Higgs mass of 110 GeV, the right-handed stop mass of 140 GeV and $\sin^2 \beta_{\text{MSSM}} = 0.8$, we find $\alpha \simeq 3 \times 10^{-2}$ and $\beta/H_* \simeq 4 \times 10^3$, leading to $h_0^2 \Omega_{\text{gw}} \simeq 2 \times 10^{-18}$. Notice that one is not allowed to lower too much the right-handed stop mass because the thermal squared mass for the stop itself would become negative at small temperature, thus leading to a physically unacceptable stable color breaking vacuum state at zero temperature.
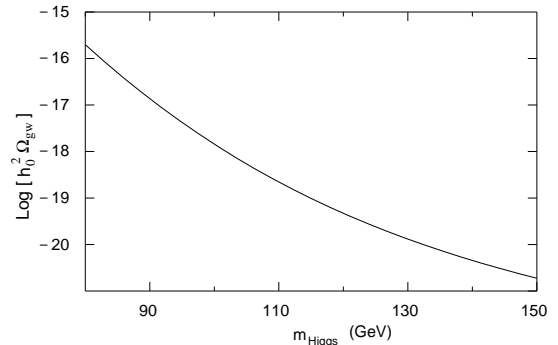
FIG. 3. $h_0^2 \Omega_{\text{gw}}$ as a function of the Higgs mass for a 155 GeV stop mass.

We have also estimated what happens if we lower the Higgs mass down to (the already excluded value of) 80 GeV, setting the right-handed stop mass at 155 GeV – which is the lower value compatible with the absence of color breaking minima – and $\sin^2 \beta = 0.8$. We obtain $\alpha \simeq 0.1$ and $\beta/H_* \simeq 2 \times 10^3$, giving $h_0^2 \Omega_{\text{gw}} \simeq 2 \times 10^{-16}$, a signal still not relevant. The situation does not improve when both Higgses are involved in the transition because the strength of the phase transition is weaker. A complete analysis of the results within the MSSM will be presented elsewhere [19]. An uncertainty in this estimate is due to the determination of v. If the phase transition is not strong enough, then v is subsonic, so that the value of $h_0^2 \Omega_{\text{gw}}$ that we have found is really an upper bound.

The situation improves considerably if we enlarge the MSSM sector adding a gauge singlet N [20]. This is the so-called Next-to-Minimal Supersymmetric Standard Model (NMSSM) and is a particularly attractive model to explain the observed baryon asymmetry at the electroweak phase transition. The relevant part of the superpotential is given by $W = \lambda H_1 H_2 N - \frac{k}{3} N^3$, where now the supersymmetric μ-parameter of the MSSM is substituted by the the combination $\lambda \langle N \rangle$, and k is a free parameter. The corresponding Higgs potential reads $V = V_F + V_D + V_{\text{soft}}$, where

$$V_F = |\lambda|^2 \left[|N|^2 (|H_1|^2 + |H_2|^2) + |H_1 H_2|^2 \right]$$
$$+ k^2 |N|^4 - (\lambda k^* H_1 H_2 N^{2*} + \text{h.c.}),$$
$$V_D = \frac{g^2 + g'^2}{8} (|H_2|^2 - |H_1|^2)^2 + \frac{g^2}{2} |H_1^\dagger H_2|^2,$$
$$V_{\text{soft}} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + m_N^2 |N|^2$$
$$- \left(\lambda A_\lambda H_1 H_2 N - \frac{1}{3} k A_k N^3 + \text{h.c.} \right). \quad (5)$$

The presence of the cubic supersymmetry breaking soft terms proportional to the parameters A_λ and A_k already at zero temperature makes it clear that within the NMSSM it is quite easy to get a very strong first-order phase transition at the electroweak scale [14]. The order of the transition is determined by these trilinear soft terms rather than by the cubic term appearing in the finite temperature one-loop corrections and the preservation of baryon asymmetry after the phase transition is

possible for masses of the lightest scalar up to about 170 GeV. Barring the possibility that the transition occurs along CP-violating directions, the potential becomes a function of three real scalar fields $(\mathrm{Re}N, \mathrm{Re}H_1, \mathrm{Re}H_2)$. In our numerical analysis we have made use of the tree-level potential (5) plus the one-loop corrections appearing at finite temperature. Overall, we have six free parameters: the coupling parameters λ and k; the soft-breaking mass terms A_λ and A_k; the zero-temperature vacuum expectation value of the singlet x and $\tan\beta_{\mathrm{MSSM}}$.

Given a set of parameters, the strategy has been again to determine the nucleation temperature T_* and the parameters α and β which in turn determine the intensity and the frequency of the stochastic gravitational background. In our analysis we have focussed on those regions of the parameter space which previous studies have shown to give rise to a large enough baryon asymmetry.
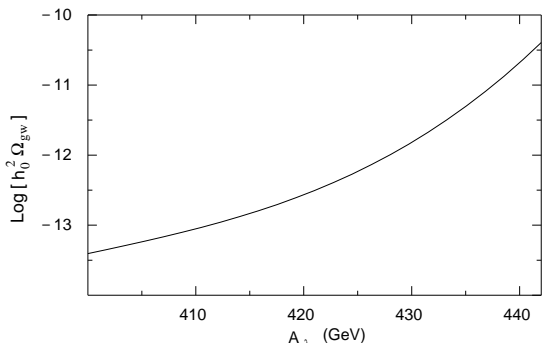

FIG. 4. $h_0^2\Omega_{\mathrm{gw}}$ as a function of A_λ for $A_k = 480$ GeV, $x = 350$ GeV, $\lambda = 0.83$, $k = 0.67$ and $\tan\beta_{\mathrm{MSSM}} = 2$
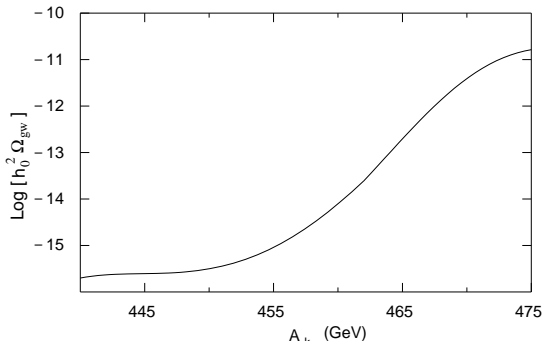

FIG. 5. $h_0^2\Omega_{\mathrm{gw}}$ as a function of A_k for $A_\lambda = 450$ GeV, $x = 350$ GeV, $\lambda = 0.83$, $k = 0.67$ and $\tan\beta_{\mathrm{MSSM}} = 2$

Typical results are summarized in Figs. 4 and 5, where we plot $h_0^2\Omega_{\mathrm{gw}}$ as a function of A_λ and A_k, respectively, with $A_k = 480$ GeV in Fig. 4 and $A_\lambda = 450$ GeV in Fig. 5. Note also that, since the transition in the NMSSM is strongly first order, it is correct to use the expression for $v(\alpha)$ computed for detonation bubbles [9].

The intensity of the produced gravity waves rapidly grows as a function of the soft breaking mass terms. This is explained by the fact that $h_0^2\Omega_{\mathrm{gw}}$ scales roughly as α^3/β^2 and a mild increase of α and decrease of β leads

to a rapid growth of the intensity. However, A_λ, A_k cannot be increased beyond the values shown in the figure because for higher values the condition $S_3(T_*)/T_* \simeq 140$ is never satisfied and the transition does not take place.

The conclusion is that values $h_0^2\Omega_{\mathrm{gw}} \simeq 4 \times 10^{-11}$ are reachable in the NMSSM in some regions of the parameter space (we have found similar values of $h_0^2\Omega_{\mathrm{gw}}$ also in other regions of the parameter space [19]). A background of this intensity would be within the reach of LISA.

We find interesting that the very same supersymmetric phase transition which provides us with a mechanism to generate the observed baryon asymmetry might also be the source of a sizeable background of gravitational waves. If such background is detected, it will be not only an indication that supersymmetry might play a role at the electroweak phase transition, but even a way to discriminate among supersymmetric models with different Higgs sectors.

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